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AN ARCHITECTURAL MODEL FOR THE ACCESS AREA/BACKBONE ALLOCATION --ETC(U)
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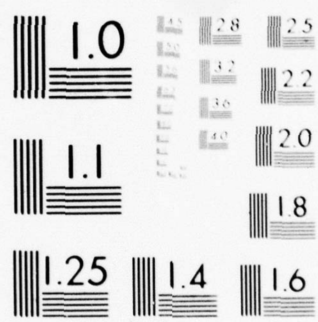

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TECHNICAL NOTE NO. 26-77

AN ARCHITECTURAL MODEL FOR THE ACCESS AREA/BACKBONE ALLOCATION PROBLEM

DECEMBER 1977

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20. ABSTRACT *(Continue on reverse side if necessary and identify by block number)*

This report develops an architectural model for performing a macroscopic design/analysis of terrestrial voice communication networks. The analysis is based on the relationships between average properties of networks: geographical area to be serviced, number of switches and the average area served by a switch, number of backbone links and the average number of links homed on a typical switch, the average capacity and length of backbone links, the number of user installations, the average length and capacity of access links, and

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the average offered load to each access link. Other important parameters are a total budget constraint and allocation criteria for expending the budget in access area design, backbone switches, and backbone links. These last parameters have a significant impact on the end to end performance of the network under routine conditions as well as the survivability of the network.

The tools developed herein are not intended for detailed network design problems - rather they are used for analysis of the average properties of large scale networks. With this understanding the examples chosen for analysis in this report approximate the CONUS AUTOVON system. The important features of this system (number of switches, backbone trunks, access area/backbone design budgets, performance and survivability) are analyzed to a gratifying degree of accuracy with our tools considering their averaging nature.

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Prepared by:

- W. P. Dotson

Approved for Publication:

W Z Chadwell

W. L. CHADWELL
Chief, Systems Engineering Division

FOREWORD

The Defense Communications Engineering Center (DCEC) Technical Notes (TN's) are published to inform interested members of the defense community regarding technical activities of the Center, completed and in progress. They are intended to stimulate thinking and encourage information exchange; but they do not represent an approved position or policy of DCEC, and should not be used as authoritative guidance for related planning and/or further action.

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Director
Defense Communications Engineering Center
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Reston, Virginia 22090

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I. INTRODUCTION

This report is the third in a planned series of six. This series, when finished, will contain the analytical techniques developed for handling technical aspects of DCS architectural planning. By architectural planning we mean the analysis of such issues as:

a. How many separate communication networks should be built to serve the many different user communities of the DCS?

b. What communication services should be provided to which user community on each network?

c. What technologies are appropriate for each network?

A key element in the analysis of these issues is a fundamental decision on the question: For what purpose is the communication service provided?

The philosophical aspects of these questions are developed in reference [1] which can be seen as a research prospectus for the DCS technical planning problem. Our principal conclusions in that report were:

a. The relative worth of a DCS alternative should be measured according to the level of service it can provide under <u>both</u> peacetime and wartime conditions. Different alternatives can then be compared when designed to the same cost constraint.

b. Network Analysis is the generic analytic tool which is

best suited for the necessary measurement/design of DCS alternatives.

c. The problem of allocating a fixed total budget into sub-budgets for voice/data networks, terrestrial/satellite network, backbone/access area networks, etc., is a pivotal one.

Another important conclusion, not explicitly stated in [1], is that traditional network analysis tools cannot be easily applied to the technical aspects of DCS architectural planning. The reason is that they require large amounts of detailed data on the constituent hardware elements of a communication network and the traffic offered to the network. This in turn implies considerable supporting efforts to generate the detailed data, and slow turnaround on evaluating alternatives.

Our basic premise regarding DCS architectural studies is that we should have the ability to analyze a very large number of alternatives. Since neither computer time nor supporting efforts are readily available to do this using traditional network analysis tools, we have set out to develop a new class of network analysis tools which require less supporting data and give faster turnarounds. We call them Architectural Models. These new tools do not perform a detailed network analysis; rather they look at the network in a macroscopic way. The analysis is based on the relationships between average properties of networks; e.g., number of switches and the average area served by a switch, the number of interswitch trunks and the average number of trunks homed on a typical switch, the average capacity and length of an interswitch

trunk, the average offered load to a typical switch, etc. Another important parameter in the analysis is the way in which a total backbone budget is allocated to a backbone switches budget and a backbone links budget and its effect on the survivability of the network. These factors are analyzed in detail in reference [2] which treats the backbone network design problem.

This report is an extension of the results in reference [2]. While we recognize that DCA does not control how money is spent in the access area (this is determined by the individual users), it has long been known that the overall network performance is enhanced when a proper balance is struck between access area designs and backbone designs. Traditional network analysis tools can and have been used to find optimum allocations of end to end blocking probability into backbone blocking probability and access area blocking probability. This report brings this issue into sharp analytical focus from a different perspective and provides a tool for quantitative assessment of this "proper balance" from a budgetary point of view. We caution the reader that this "proper balance" issue is not a separable one; that is, the analysis herein shows that the access area and the backbone should be designed each in the light of the other. Thus, in a way this report not only goes counter to current practice in the way it would allocate a budget - it also goes counter to the conventional technical wisdom which would have us design the access area and the backbone independently, each to some required blocking probability.

1. PROBLEM DISCUSSION

The communications network problem under consideration is shown in Figure 1. In the figure, circles represent backbone switches (nodes), squares represent local switches, and triangles represent terminal devices. Interconnecting lines may represent local links, access area links, or backbone links, as indicated in the illustration. Note that three categories of traffic are identified in Figure 1:

a. Local traffic between terminal devices, which represents a load only on local links and the local switch.

b. Intra-access area traffic, which represents a load on local links, local switches, access area links, and the backbone node.

c. Backbone traffic, which represents a load on local links, local switches, access area links, backbone nodes, and backbone links.

This report will not consider local traffic since it loads only resources which are not properly a part of the DCS. Rather, we will consider the intra-access and backbone traffic offered to the access area/backbone network by the local switches. The numbers and locations of local switches, the load offered by each switch, and the load destination then become important parameters in our problem. We now lay out the specific assumptions required.

An access area/backbone communications network is to be designed to service a rectangular area whose dimensions are A and B miles, as shown in Figure 2. Very little is known about the offered traffic load the network must handle. Consequently, using the Laplacian assumption of rationality [3], we assume that:

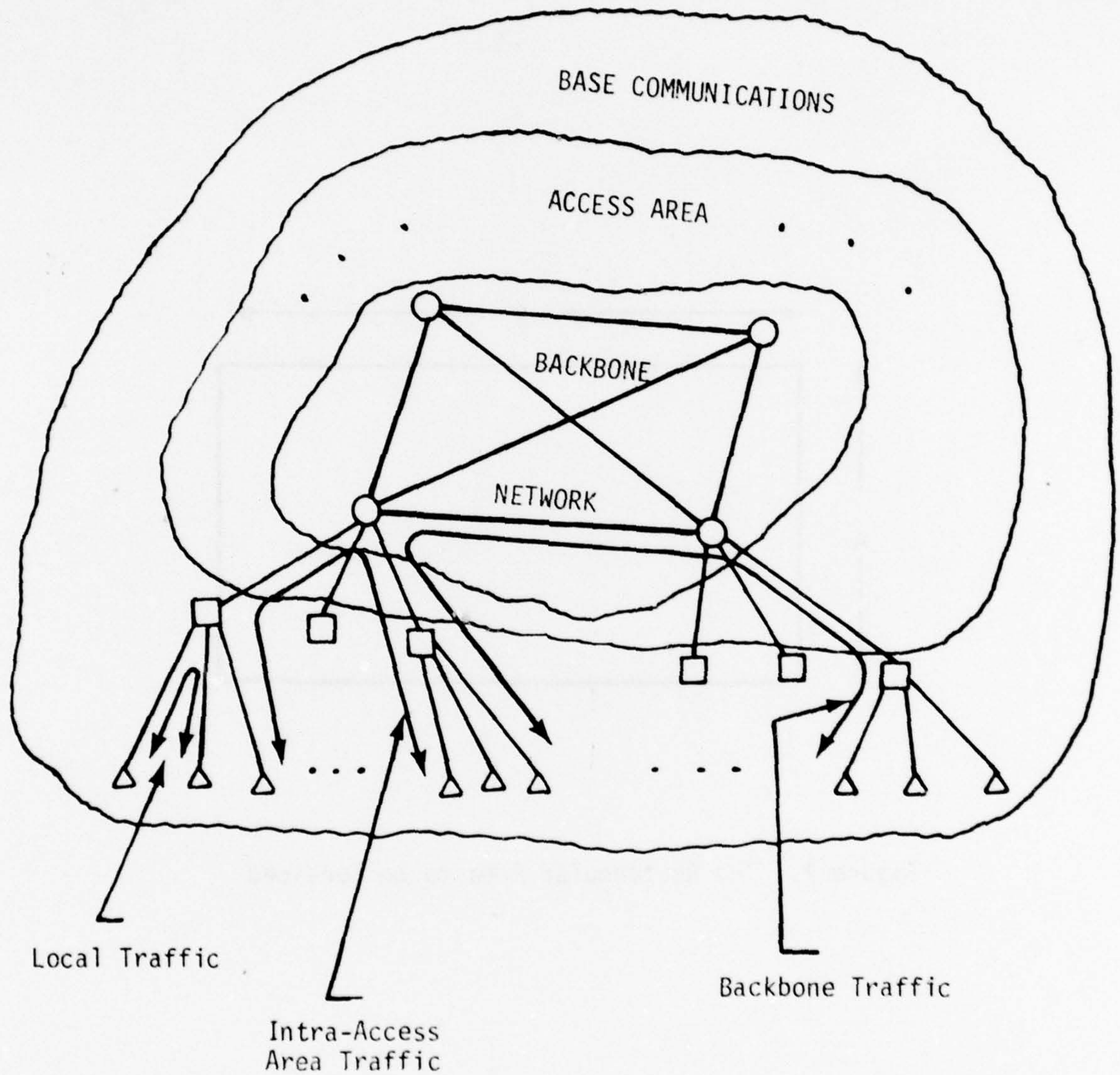


Figure 1. The Communications Network Problem

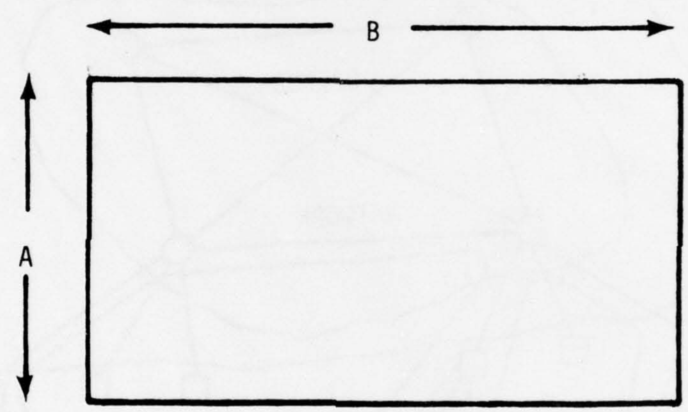


Figure 2. The Rectangular Area to be Serviced

A1. There are K local switches uniformly distributed in the area and all switches are equally likely to generate traffic.

A2. A randomly selected local switch is equally likely to place a call to any one of the other switches in the area as to any other.

A3. The total offered traffic (intra-access area and backbone) for the network is E erlangs.

For our purposes, it is reasonable to assume that the network is constructed of three basic elements: backbone nodes, backbone links, and access area links. Local switches need not be explicitly considered, since by assumption A1, their total cost contribution will be identical for all designs. A cost model for each element under consideration is given by the assumptions:

A4. Individual backbone node costs, D_n, comprise a fixed cost, a, and a cost per channel termination, b:

$$D_n = a + bt$$

where t is the number of backbone channels terminating on the node.

A5. A given backbone link cost, D_{bL}, is a function of the number of channels, c, in the link; and the length in miles, ℓ, of the link:

$$D_{bL} = kc\ell$$

where k is the cost per channel mile. All links are assumed to be full duplex.

A6. Access area link costs, D_{aL}, comprise a charge per channel mile and termination charges:

$$D_{aL} = rcd + sc$$

where

r : cost per channel mile

c : number of channels

d : length, in miles, of the access link

s : termination charge per channel.

Our approach to this problem requires that we impose some geometric regularity on potential network designs while leaving as design variables the number of backbone nodes; the number, length, and capacity of backbone links; and the length and capacity of access area links. Thus, if we were to investigate a design consisting of MxN backbone nodes, they would be arranged in a uniform grid inside the AxB area, as shown in Figure 3. Each backbone node would service a single access area; a blowup of one such access area is also shown in Figure 3.

Our problem is to find an optimum design in the above parameters. By optimum we mean that design, costing no more than a fixed amount, D, which maximizes the network throughput, but which also must provide a minimum acceptable throughput after removal of a designated number of backbone nodes. This problem was solved, for the backbone design only, in reference [2]. This report extends the results of reference [2] by addressing the problem of allocating a fixed total budget, D, between access area and backbone designs.

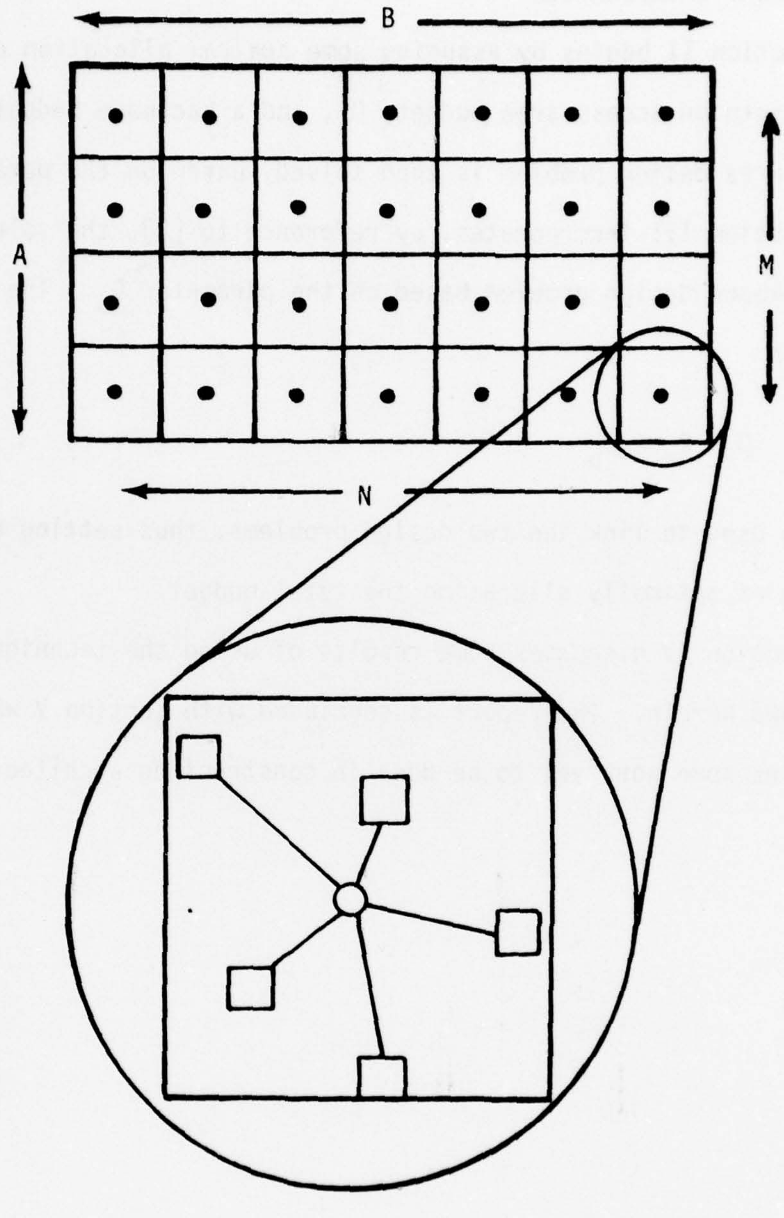


Figure 3. Representative Backbone Node Array and Access Area

2. REPORT ORGANIZATION

Section II begins by assuming some *apriori* allocation of the total budget into an access area budget, D_a, and a backbone budget, D_b. The access area design problem is then solved, based on the parameter D_a.

Section III incorporates, by reference to [2], the solution of the backbone design problem based on the parameter D_a. The obvious relation

$$D = D_a + D_b$$

is then used to link the two design problems, thus setting up the new problem of optimally allocating the total budget.

Section IV discusses some results of using the techniques developed herein. The report is concluded with section V which indicates some work yet to be done in constructing architectural models.

II. ACCESS AREA MODEL

This section develops the models which describe access area network performance. The implicit assumption is that access area design is based on a star configuration. That is, all local switches in the access area serviced by a particular backbone node are homed directly on that node.

1. TURNAROUND TRAFFIC

The total offered traffic, E, considered in this report is intra-access area offered traffic, E_a, and backbone offered traffic, E_b. We assume a local switch offering its share of the total traffic to a backbone node where part of the offering is "turned around." The "turnaround traffic" is E_a. The remainder is E_b. Thus,

$$E = E_a + E_b. \tag{1}$$

By our assumptions on the traffic distribution, the backbone nodes will see an intra-access area traffic offering of

$$E_a = E/MN. \tag{2}$$

Similarly,

$$E_b = E\left(\frac{MN-1}{MN}\right). \tag{3}$$

Thus, the ratio of intra-access area offered traffic to total offered traffic is 1/MN.

2. INTRA-ACCESS AREA CARRIED TRAFFIC

We assume, temporarily, that an *apriori* decision has been made to spend D_a dollars in all access area designs. There are MN access areas to be designed, and each is a square, as shown in Figure 3, with dimension u [2]; therefore,

$$u = [AB/MN]^{1/2} . \tag{4}$$

From equation (4) and assumption A1 it is shown in [4] and [5] that the average length of an access area link, d, is

$$d = 0.381\, u . \tag{5}$$

Since there are K local switches to be linked to backbone nodes, and D_a dollars available for all access area links, the average amount that can be spent on each access area link can be determined from the access area link cost model, D_{aL}, by the relation:

$$D_{aL} = D_a/K = rcd + sc . \tag{6}$$

From (6) the average access area link capacity is c ; i.e.,

$$c = D_a/K(rd + s) . \tag{7}$$

The total offered load to each access area link is, on the average, ρ, where

$$\rho = E/K , \tag{8}$$

since access area links service both intra-access area and backbone traffic.

From (7), (8), and (2) a good approximation can be developed for the intra-access area carried traffic, E_{ac}:

$$E_{ac} = (1-q_a)^2 E_a = (1-q_a)^2 \frac{E}{MN} \tag{9}$$

where q_a is the blocking probability of the average access area link and is given by

$$q_a = E_B(\rho, c) = \frac{\rho^c/c!}{\sum_{i=0}^{c} \rho^i/i!} \, . \tag{10}$$

The squared term shows up in (9), since two access area links in series are required to service an intra-access area call. Note that c in (10) may not be integer due to the averaging process used. In this event, a logarithmic interpolation formula suggested in [6] is used to calculate q_a.

III. TOTAL CARRIED TRAFFIC AND THE ALLOCATION PROBLEM

The previous section provided a model for calculating intra-access area carried traffic, E_{ac}. E_{ac} is related through a chain of equations to an assumed *apriori* allocation of D_a dollars to the access area. In a similar manner we can calculate carried backbone traffic, E_{bc}. E_{bc} can also be related to D_a. Total carried traffic, E_c, can thus be found and related to D_a. The resulting equation can then be analyzed as a function of D_a to determine an optimum allocation of the total budget, D, into D_a and D_b.

1. BACKBONE CARRIED TRAFFIC AND TOTAL CARRIED TRAFFIC

In another report [2], we developed a model for finding (backbone) network GOS, G. Those results are incorporated by reference in this report, and, for the sake of notational convenience, we rename GOS as q_b, the probability of call blocking in the backbone network. With q_b, the carried backbone traffic is E_{bc}:

$$E_{bc} = E_b(1-q_b)(1-q_a)^2 . \tag{11}$$

This equation follows from the reasoning that a successfully offered backbone call must traverse two series access area links as well as the backbone network.

Combining (3), (9), and (11) leads to an equation for total carried traffic, E_c :

$$E_c = \frac{E}{MN}(1-q_a)^2[1+(MN-1)(1-q_b)] . \tag{12}$$

Our problem now is this: For some fixed backbone architectural design (in the terminology of [2], for some fixed (I,M,N)) find the maximum value of E_c as a function of D_a. Later, we can vary (I,M,N).

2. THE ALLOCATION PROBLEM

To change E_c in (12) we must change q_a and q_b. Equations (7) and (10) show that q_a is a monotonically decreasing function of D_a. Thus, a cursory analysis shows:

a. q_a can be made smaller by increasing D_a.

b. Increasing D_a decreases D_b (since D_b=D-D_a). A secondary effect of increasing D_a is to increase the backbone offered load as seen by the backbone nodes.

c. The net result would appear to be an increase in q_b as D_a increases.

d. Thus, it is not immediately apparent whether the overall effect is good or bad.

In order to investigate this issue we shall first summarize the information developed so far in both this report and [2]. The issue is notationally complicated, so we shall also at this point introduce a set of (hopefully) mnemonic notation to be used hereafter. We have that

E_c : total carried traffic

$$E_c = \frac{E}{MN}(1-q_a)^2[1+(MN-1)(1-q_b)] \tag{13}$$

q_a : average access area link blocking probability

$$q_a = E_B(\rho,\ C_a) \tag{14}$$

ρ : average access area link offered load

$$\rho = E/K \tag{15}$$

C_a : average access area link capacity

$$C_a = c = D_a/K(rd+s). \tag{16}$$

The development of q_b is accomplished using the results of [7]. Some notational changes are required, for the following reason. In [7], E referred to backbone offered traffic <u>at the backbone nodes</u>. In this report we have backbone offered traffic <u>at the local switch</u>, E_b. Thus, the backbone offered traffic must be able to traverse two series access links before being "offered" to the backbone node. Hence, equation (62) of [7] must be rewritten here with the replacement:

$$E \leftarrow E_b(1-q_a)^2. \tag{17}$$

We have then

$$E_T' = \left[\frac{E_b(1-q_a)^2}{MN} + E_T'\left(1-\frac{1}{L}\right)\right](1-q) \tag{18}$$

where

$$q = E_B\left[\frac{\dfrac{E_b(1-q_a)^2}{MN} + E_T'\left(1-\dfrac{1}{L}\right)}{I}, C_L\right] \tag{19}$$

and

C_L : average backbone link capacity.

From equation (93) of [2], replacing D by D_b results in

$$C_L = [D_b - aMN]/[bMNI + kMNI\ell/2] . \tag{20}$$

Equation (55) of [2] is used to develop

q_b : backbone blocking probability

$$q_b = 1 - \left[E_T^{\,\prime} MN/E_b(1-q_a)^2 L\right] . \tag{21}$$

By defining the following set of constants, the preceding equations can be simplified:

$$C_1 = 1/K(rd+s) \tag{22}$$

$$C_2 = M^2N^2/(MN-1)EL \tag{23}$$

$$C_3 = E(MN-1)/M^2N^2 \tag{24}$$

$$C_4 = 1 - 1/L \tag{25}$$

$$C_5 = C_3/I \tag{26}$$

$$C_6 = C_4/I \tag{27}$$

$$C_7 = (D-aMN)/(bMNI + kMNI\ell/2) \tag{28}$$

$$C_8 = 1/(bMNI + kMNI\ell/2) . \tag{29}$$

Our working equations then become

$$E_c = \frac{E}{MN}(1-q_a)^2[1+(MN-1)(1-q_b)] \tag{30}$$

$$q_a = E_B(\rho, C_a) \quad (31)$$

$$q_b = 1 - \left[C_2 E_T'/(1-q_a)^2\right] \quad (32)$$

$$E_T' = I\rho_L[1 - q_L] \quad (33)$$

where

$$C_2 = C_1 D_a \quad (34)$$

ρ_L : average backbone link offered load

$$\rho_L = C_5(1-q_a)^2 + C_6 E_T' \quad (35)$$

$$C_L = C_7 - C_8 D_a \quad (36)$$

q_L : average backbone link blocking probability

$$q_L = E_B(\rho_L, C_L) . \quad (37)$$

Our problem now is to find that value of D_a for which E_c, given by (30), is a maximum.

3. SOLUTION OF THE ALLOCATION PROBLEM

The allocation problem can be approached using classical calculus; specifically, using differentiation of composite functions [8], we must solve the equation,

$$\frac{dE_c}{dD_a} = 0 = -\frac{E}{MN}(1-q_a)\left\{2[MN(1-q_b) + q_b]\frac{dq_a}{dD_a} + (MN-1)(1-q_a)\frac{dq_b}{dD_a}\right\} \quad (38)$$

In order to find $\frac{dq_a}{dD_a}$ and $\frac{dq_b}{dD_a}$, equations (31), (32), and (33) must be analyzed. Some auxiliary relations will come in handy later. From [9],

$$E_B(\rho, C) = \frac{\rho E_B(\rho, C-1)}{C + \rho E_B(\rho, C-1)}. \tag{39}$$

Equation (39) can be solved for $E_B(\rho, C-1)$; i.e.,

$$E_B(\rho, C-1) = \frac{CE_B(\rho, C)}{\rho - \rho E_B(\rho, C)}. \tag{40}$$

Next, we need

$$\frac{d}{d\rho} E_B(\rho, C) = \frac{d}{d\rho}\left[\frac{\rho^C/C!}{\sum_{i=0}^{C} \rho^i/i!}\right]. \tag{41}$$

After some manipulation involving the use of (40), we find

$$\frac{d}{d\rho} E_B(\rho, C) = \frac{E_B(\rho, C)}{\rho}[C - \rho + \rho E_B(\rho, C)]. \tag{42}$$

We also need the "derivative" of $E_B(\rho, C)$ with respect to C. Due to the integer nature of the parameter C, we "approximate" this by

$$\frac{d}{dC} E_B(\rho, C) = E_B(\rho, C+1) - E_B(\rho, C). \tag{43}$$

By using (39), (43) reduces to

$$\frac{d}{dC} E_B(\rho, C) = \frac{\rho E_B(\rho, C)}{C+1 + \rho E_B(\rho, C)} - E_B(\rho, C). \tag{44}$$

We can now return to the main problem. Using (44), (34), and (14), we find from (31)

$$\frac{dq_a}{dD_a} = C_1\left[\frac{\rho q_a}{C_a + 1 + \rho q_a} - q_a\right]. \tag{45}$$

From (32), we find

$$\frac{dq_b}{dD_a} = \frac{\partial q_b}{\partial E_T'}\frac{dE_T'}{dD_a} + \frac{\partial q_b}{\partial q_a}\frac{dq_a}{dD_a} = -\frac{C_2}{(1-q_a)}\frac{dE_T'}{dD_a} - \frac{2C_2E_T'}{(1-q_a)^3}\frac{dq_a}{dD_a}$$

$$= \left[C_2\frac{dE_T'}{dD_a} + 2(1-q_b)\frac{dq_a}{dD_a}\right]\left[\frac{-1}{(1-q_a)}\right]. \tag{46}$$

We already have dq_a/dD_a; we must now find dE_T'/dD_a. Tracing through equations (33), (37), (35), and (36), we see that the only variables involved are E_T' and D_a. Therefore, using the notion of implicit functions [8], we rewrite (33) as

$$h(E_T', D_a) \equiv 0 = E_T' - I\rho_L[1-q_L]. \tag{47}$$

Since $h(E_T', D_a)$ is identically zero, we have

$$\frac{dh}{dD_a} \equiv 0 = \frac{\partial h}{\partial E_T'}\frac{dE_T'}{dD_a} + \frac{\partial h}{\partial D_a}, \tag{48}$$

or

$$\frac{dE_T'}{dD_a} = -\frac{\partial h/\partial D_a}{\partial h/\partial E_T'}. \tag{49}$$

Now, we must find $\partial h/\partial D_a$ and $\partial h/\partial E_T'$.

From (47),

$$\frac{\partial h}{\partial D_a} = - I \frac{\partial \rho_L}{\partial D_a} + I\rho_L \frac{\partial q_L}{\partial D_a} + Iq_L \frac{\partial \rho_L}{\partial D_a} = - I \frac{\partial \rho_L}{\partial D_a}(1-q_L) + I\rho_L \frac{\partial q_L}{\partial D_a}. \quad (50)$$

From (35),

$$\frac{\partial \rho_L}{\partial D_a} = -2C_5 \frac{\partial q_a}{\partial D_a}(1-q_a). \quad (51)$$

Using (45), (51) becomes

$$\frac{\partial \rho_L}{\partial D_a} = -2C_1C_5 \left[\frac{\rho q_a}{C_a+1+\rho q_a} - q_a\right](1-q_a). \quad (52)$$

The next item, using (37), is

$$\frac{\partial q_L}{\partial D_a} = \frac{\partial E_B(\rho_L, C_L)}{\partial \rho_L}\frac{\partial \rho_L}{\partial D_a} + \frac{\partial E_B(\rho_L, C_L)}{\partial C_L}\frac{\partial C_L}{\partial D_a}. \quad (53)$$

Using (42), (35), (44), and (36), (53) becomes

$$\frac{\partial q_L}{\partial D_a} = \frac{q_L}{\rho_L}[C_L-\rho_L+\rho_L q_L]\left[-2C_5(1-q_a)\frac{\partial q_a}{\partial D_a}\right] + \left[\frac{\rho_L q_L}{C_L+1+\rho_L q_L} - q_L\right](-C_8). \quad (54)$$

Now, using (45), (54) becomes

$$\frac{\partial q_L}{\partial D_a} = \frac{q_L}{\rho_L}[C_L-\rho_L+\rho_L q_L][-2C_1C_5(1-q_a)]\left[\frac{\rho q_a}{C_a+1+\rho q_a} - q_a\right]$$

$$-C_8\left[\frac{\rho_L q_L}{C_2+1+\rho_L q_L} - q_L\right]. \quad (55)$$

This equation can be simplified to

$$\frac{\partial q_L}{\partial D_a} = \frac{2C_1C_5q_aq_Lr_L(1-q_a)}{\rho_L}\left[\frac{r_a+1}{r_a+1+\rho}\right] + C_8q_L\left[\frac{r_L+1}{r_L+1+\rho_L}\right] \quad (56)$$

where

r_a : residual capacity of the average access area link

$$r_a = C_a - \rho(1-q_a) \tag{57}$$

r_L : residual capacity of the average backbone link

$$r_L = C_L - \rho_L(1-q_L). \tag{58}$$

Similarly, (52) can be rewritten as

$$\frac{\partial \rho_L}{\partial D_a} = 2C_1C_5q_a(1-q_a)\left[\frac{r_a+1}{r_a+1+\rho}\right]. \tag{59}$$

Substituting (58) and (59) into (50) results in

$$\frac{\partial h}{\partial D_a} = 2IC_1C_5R_aq_a(1-q_a)[q_Lr_L-(1-q_L)] + IC_8q_L\rho_LR_L \tag{60}$$

where

$$R_a = \frac{r_a+1}{r_a+1+\rho} \tag{61}$$

and

$$R_L = \frac{r_L+1}{r_L+1+\rho_L}\,. \tag{62}$$

Returning now to (47)

$$\frac{\partial h}{\partial E_T'} = 1+I\rho_L\frac{\partial q_L}{\partial E_T'} - I(1-q_L)\frac{\partial \rho_L}{\partial E_T'}\,. \tag{63}$$

From (35) we see that

$$\frac{\partial \rho_L}{\partial E_T'} = C_6. \tag{64}$$

From (37)

$$\frac{\partial q_L}{\partial E_T'} = \frac{\partial E_B(\rho_L,C_L)}{\partial \rho_L}\frac{\partial \rho_L}{\partial E_T'} + \frac{\partial E_B(\rho_L,C_L)}{\partial C_L}\frac{\partial C_L}{\partial E_T'} . \tag{65}$$

Using (42), (64), (44), and (36) , we see that

$$\frac{\partial q_L}{\partial E_T'} = C_6 \frac{q_L}{\rho_L}[C_L-\rho_L+\rho_L q_L] . \tag{66}$$

Substituting (66) and (64) into (63) ,

$$\frac{\partial h}{\partial E_T'} = 1 + I\, C_6 q_L[C_L-\rho_L+\rho_L q_L] - IC_6(1-q_L) \tag{67}$$

or

$$\frac{\partial h}{\partial E_T'} = 1 + IC_6[q_L r_L-(1-q_L)]. \tag{68}$$

Equations (60) and (68) may now be substituted into (49) to find

$$\frac{dE_T'}{dD_a} = - \frac{2IC_1C_5R_a q_a(1-q_a)[q_L r_L-(1-q_L)]+IC_8 q_L\rho_L R_L}{1+IC_6[q_L r_L-(1-q_L)]} . \tag{69}$$

Now we can substitute (45) and (69) into (46) to find, using (61),

$$\frac{dq_b}{dD_a} = \frac{C_2}{(1-q_a)}\left\{\frac{2IC_1C_5R_a q_a(1-q_a)[q_L r_L-(1-q_L)]+IC_8 q_L\rho_L R_L}{1+IC_6[q_L r_L-(1-q_L)]} + \frac{2C_1}{C_2} q_a(1-q_b)\, R_a\right\} . \tag{70}$$

Equations (45) and (70) can now be substituted into (38), again using (61), to find

$$\frac{dE_c}{dD_a} = -\frac{E}{MN}(1-q_a)\left\{2[MN(1-q_b)+q_b](-C_1 q_a R_a)\right.$$

$$\left. +C_2(MN-1)\left[\frac{2IC_1C_5R_a q_a(1-q_a)[q_L r_L-(1-q_L)]+IC_8 q_L \rho_L R_L}{1+IC_6[q_L r_L-(1-q_L)]} + \frac{2C_1}{C_2} q_a(1-q_b) R_a\right]\right\} . \quad (71)$$

To solve our allocation problem, we must find D_a such that

$$\frac{dE_c}{dD_a} = 0 . \quad (72)$$

4. A SEARCH ALGORITHM FOR THE ALLOCATION PROBLEM

A cursory examination of equation (71) shows that it is transcendental; hence, it cannot be solved for its zero(s) in closed form. To begin the construction of an algorithm to find the zero(s), we rewrite (71), set equal to zero, as

$$C_1 q_a R_a = \frac{C_2(MN-1)}{2[MN(1-q_b)+q_b]}\left\{\frac{2IC_1C_5R_a q_a(1-q_a)[q_L r_L-(1-q_L)]}{1+IC_6[q_L r_L-(1-q_L)]}\right.$$

$$\left. + \frac{IC_8 q_L \rho_L R_L}{1+IC_6[q_L r_L-(1-q_L)]} + \frac{2C_1}{C_2} q_a(1-q_b)R_a\right\} . \quad (73)$$

Dividing through by $q_a R_a$ and rearranging,

$$C_1 - \frac{C_2(MN-1)IC_8 q_L \rho_L R_L}{2[MN(1-q_b)+q_b]\{1+IC_6[q_L r_L-(1-q_L)]\}q_a R_a} = \left[\frac{MN-1}{MN(1-q_b)+q_b}\right] \cdot$$

$$\left[\frac{IC_1C_2C_5(1-q_a)[q_L r_L-(1-q_L)]+C_1(1-q_b)\{1+IC_6[q_L r_L-(1-q_L)]\}}{1+IC_6[q_L r_L-(1-q_L)}\right] . \quad (74)$$

We need to clarify the notation. Let:

$$K_b = [MN(1-q_b)+q_b] \tag{75}$$

$$K_L' = q_L r_L - (1-q_L) \tag{76}$$

$$K_L = 1+IC_6 K_L' . \tag{77}$$

Using these, (74) becomes

$$C_1 - \frac{C_2(MN-1)IC_8 q_L \rho_L R_L}{K_b K_L q_a R_a} = \frac{MN-1}{K_b}\left[\frac{IC_1C_2C_5(1-q_a)K_L'+C_1(1-q_b)K_L}{K_L}\right] \tag{78}$$

or

$$\frac{C_1K_bK_L q_a R_a - C_2(MN-1)IC_8 q_L\rho_L R_L}{q_a R_a} = (MN-1)\{IC_1C_2C_5(1-q_a)K_L'+C_1(1-q_b)K_L\} . \tag{79}$$

Rearranging (79) to isolate access area terms

$$C_1K_bK_L - (MN-1)\{IC_1C_2C_5(1-q_a)K_L'+C_1(1-q_b)K_L\} = \frac{C_2(MN-1)IC_8 q_L\rho_L R_L}{q_a R_a} \tag{80}$$

or

$$\frac{1}{q_a R_a} = \frac{C_1K_bK_L-(MN-1)\{IC_1C_2C_5(1-q_a)K_L'+C_1(1-q_b)K_L\}}{IC_2C_8(MN-1)q_L\rho_L R_L} . \tag{81}$$

Equation (81) will form the basis for the search algorithm of this subsection. More specifically, (81) can be rewritten as three equations:

$$X(D_a) = 1/q_a R_a \tag{82}$$

$$Y(D_a) = \frac{C_1K_bK_L-(MN-1)\{IC_1C_2C_5(1-q_a)K_L'+C_1(1-q_b)K_L\}}{IC_2C_8(MN-1)q_L\rho_L R_L} \tag{83}$$

$$X(D_a^*) - Y(D_a^*) = 0 . \tag{84}$$

The algorithm will be structured to find D_a^* such that

$$|X(D_a^*) - Y(D_a^*)| \leq \epsilon \rightarrow 0. \tag{85}$$

To understand the algorithm, we need first to indicate that an extreme point analysis of (82) and (83) results in:

$$X(0) \simeq (K+E)/E \tag{86}$$

$$X(D) \simeq \infty \tag{87}$$

$$Y(0) \simeq \infty \tag{88}$$

$$Y(D) \simeq EL(b+k\ell/2)/MNK(rd+s). \tag{89}$$

A hypothetical plot of (82) and (83) based on (86) - (89) is shown in Figure 4. *The figure suggests (but does not prove) that equation (71)* has only one zero. It is not our intention to provide a proof for this conjecture. Certainly, if it is not true, our algorithm, shown in Figure 5, will not work reliably and we can find a counter example to the conjecture by running some design problems through the algorithm.

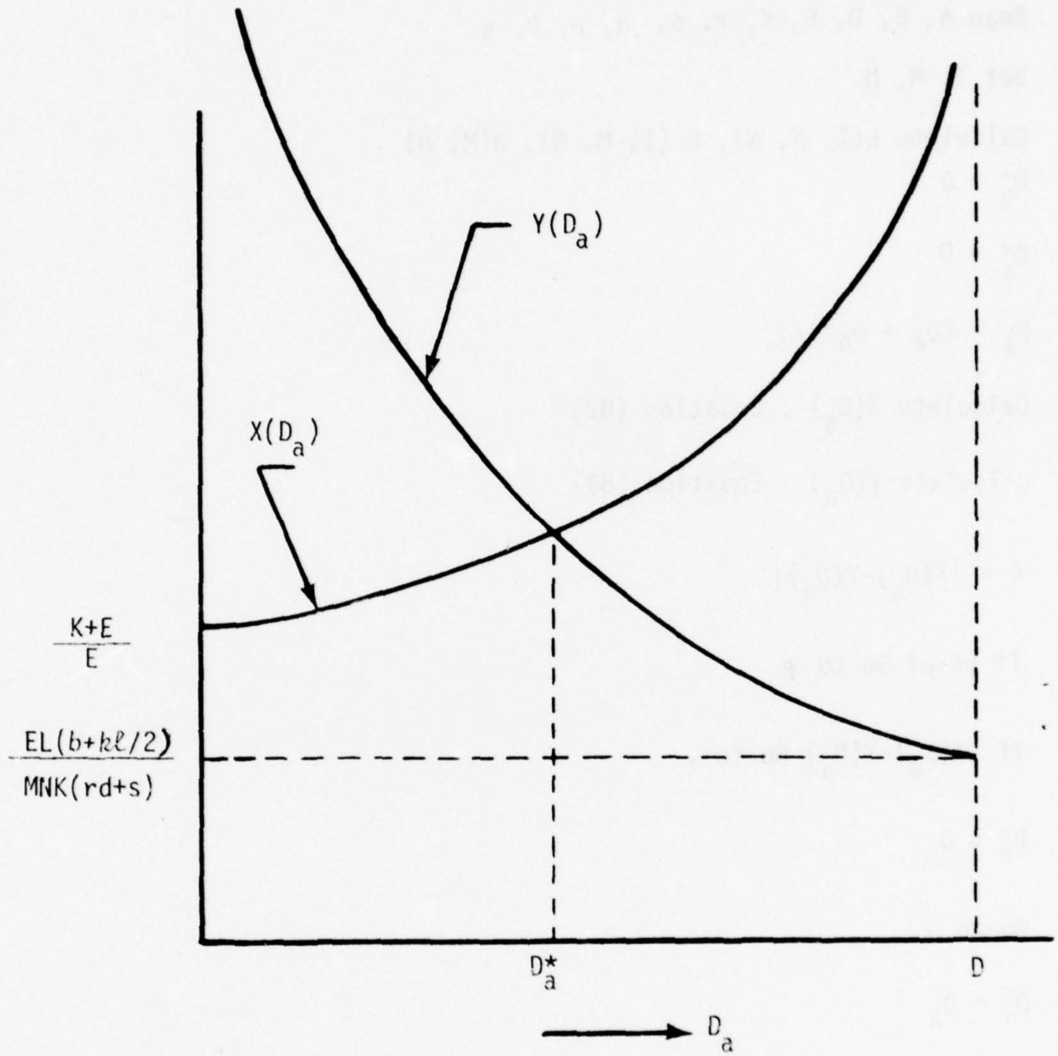


Figure 4. Hypothetical Plot of Equations (82) and (83)

Read A, B, D, E, K, r, s, a, b, k, ε

Set I, M, N

Calculate L(I, M, N), ℓ (I, M, N), d(M, N)

$D_a' = 0$

$D_a'' = D$

α $D_a = (D_a' + D_a'')/2$

Calculate $X(D_a)$: Equation (82)

Calculate $Y(D_a)$: Equation (83)

$\delta = |X(D_a)-Y(D_a)|$

If $(\delta \leq \varepsilon)$ Go to β

If $(X(D_a)<Y(D_a))$ Go to γ

$D_a'' = D_a$

Go to α

γ $D_a' = D_a$

Go to α

β $D_a^* = D_a$

STOP

Figure 5. An Algorithm for Computing D_a^*

IV. SUMMATION AND RESULTS

The preceding sections of this report have developed some mathematical models which, while not terribly difficult, are fairly intricate. This is particularly so due to the incorporation, by reference, of the results of [2] and the concomitant difficulties of changes in notation from that report to this one. For that reason, this section will be devoted to two areas: The first area will recapitulate major results of [2] and this report, and also develop an algorithm to graphically display the principal thrust of this report; viz., there exists an optimum allocation of a total budget, D, into an access area budget, D_a, and a backbone budget, $D_b = D - D_a$. As an example to show this, we present a plot of E_c vs D_a. The second area will develop an algorithm based on the results of [2] and this report. This algorithm will be used to develop the performance characteristic of optimum <u>terrestrial</u> (access area/backbone) designs.

1. RECAPITULATION AND E_c vs D_a

The exogenous variables in the terrestrial network design problem are:

A : width, in miles, of the rectangular area to be serviced.

B : length, in miles, of the rectangular area to be serviced

D : the total amount to be spent on the design

E : the total network offered traffic in erlangs

K : the total number of local switches

a : fixed cost per backbone node

b : cost per backbone link channel termination on a backbone node

k : cost per channel mile of a backbone link

r : cost per channel mile of an access area link

s : termination charge per access area link channel.

The endogenous variables in the terrestrial network design problem are:

D_a : amount to be spent on the access area design

D_b : amount to be spent on the backbone design

E_a : intra-access area offered traffic in erlangs

E_b : backbone offered traffic in erlangs

I : average incidence degree of a backbone node

M : number of backbone nodes along the A dimension of the rectangular area to be serviced

N : number of backbone nodes along the B dimension of the rectangular area to be serviced.

The two major design variables are:

L(I,M,N) : number of tandem backbone links used in placing the average backbone call over the most direct route

ℓ(I,M,N) : the length, in miles, of the average backbone link.

Linking variables are:

c (or C_a) : capacity, or number of channels, of the average access area link

$$C_a = D_a / K(rd + s) \tag{90}$$

where

$$d = d(M,N) = .381[AB/MN]^{\frac{1}{2}}. \tag{91}$$

ρ : offered load, in erlangs, to the average access area link

$\rho = E/K$

q_a : blocking probability of the average access area link

$$q_a = E_B\ (\rho,\ C_a) \tag{93}$$

E_{ac} = intra-access area carried traffic

$$E_{ac} = (1-q_a)^2 \frac{E}{MN}\ . \tag{94}$$

c(or C_L) : capacity, or number of channels, of the average backbone link

$$C_L = C_7 - C_8\ D_a \tag{95}$$

where

$$C_7 = (D-aMN)/(bMNI + k\,MNI\ell/2) \tag{96}$$

$$C_8 = 1/(bMNI + k\,MNI\ell/2) \tag{97}$$

$$\ell = \ell(I,M,N)\ ;\ \text{See ref. [2].} \tag{98}$$

ρ_L : offered load, in erlangs, to the average backbone link

$$\rho_L = C_5(1-q_a)^2 + C_6 E_T \tag{99}$$

where

$$C_5 = E(MN-1)/IM^2N^2 \tag{100}$$

$$C_6 = (1-{}^1/_L)/I \tag{101}$$

$$L = L(I,M,N)\ ;\ \text{see ref. [2]} \tag{102}$$

$$E_T' = I\rho_L[1-q_L]\ ;\ \text{see ref [2]} \tag{103}$$

q_L : blocking probability of the average backbone link

$$q_L = E_B(\rho_L, C_L). \tag{104}$$

q_b : backbone blocking probability

$$q_b = 1-\left[C_2\ E_T'/(1-q_a)^2\right] \tag{105}$$

where

$$C_2 = M^2N^2/(MN-1)EL \tag{106}$$

E_{bc} : backbone carried traffic

$$E_{bc} = \frac{E(MN-1)}{MN}(1-q_b)(1-q_a)^2 . \tag{107}$$

The optimization variables are D_a, I, M, and N. For a fixed (I,M,N) we have the remaining optimization variable, D_a, which links the access area/backbone design problem through the relation

$$D = D_a + D_b . \tag{108}$$

By varying D_a, we can vary

E_c : total carried traffic

$$E_c = E_{ac} + E_{bc} . \tag{109}$$

An algorithm for computing E_c vs D_a is given in Figure 6. The results of exercising the algorithm on a problem with the data shown below are given in Figure 7.

$A = 2,000$

$B = 3,000$

$D = 10,000,000$

$E = 6000$

$K = 1500$

$a = 8000$

$b = 106$

$k = 0.5$

$r = 0.5$

$s = 106$

$I = 14$

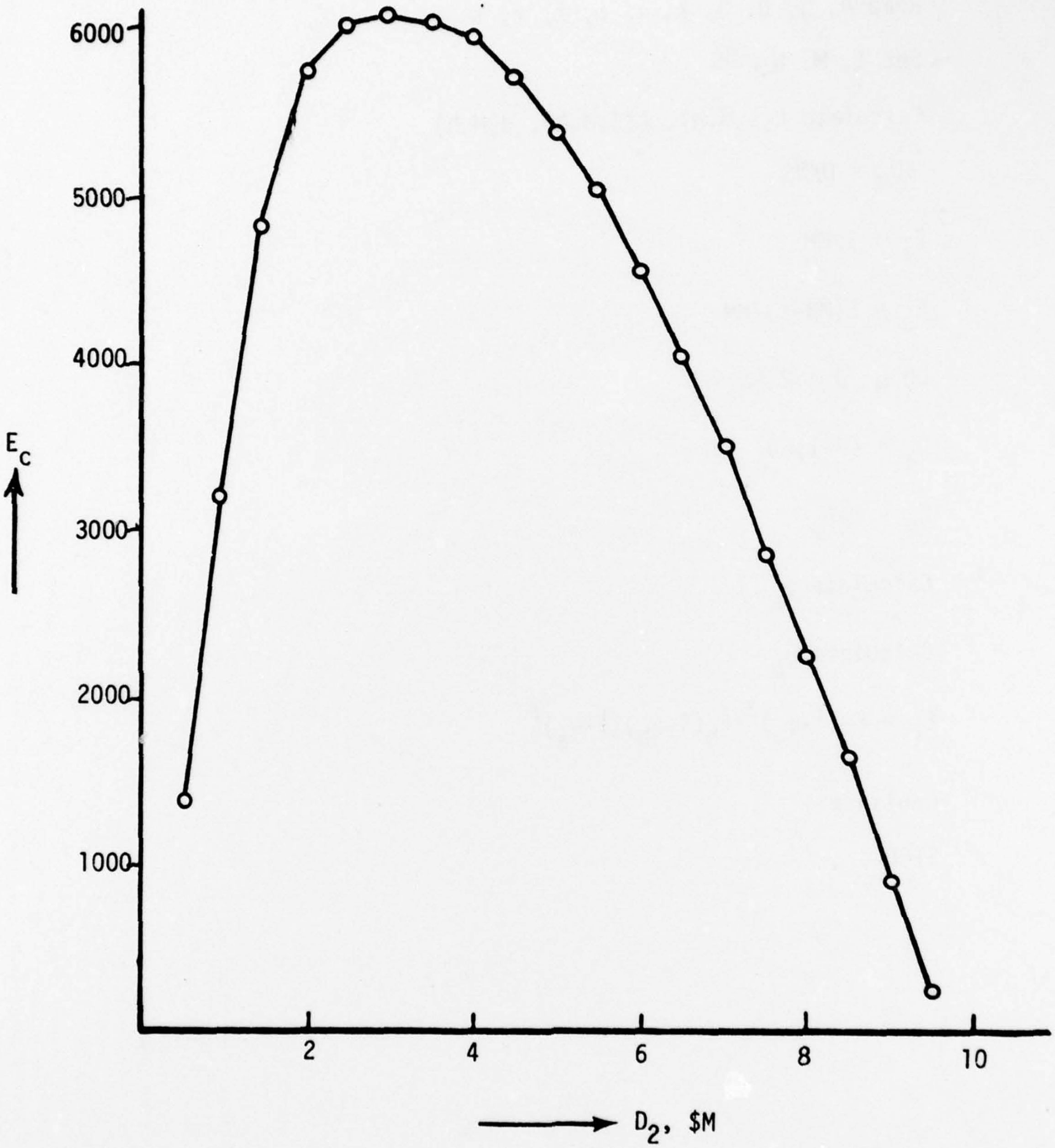
$M = 6$

$N = 9$.

```
     Read A, B, D, E, K, a, b, k, r, s
     Set I, M, N , NS
     Calculate L(I,M,N), ℓ(I,M,N), d(M,N)
     ΔD_a = D/NS
     E_a = E/MN
     E_b = E(MN-1)/MN
     DO α  J = 2,NS
     D_a = (J-1)ΔD_a
     D_b = D-D_a
     Calculate q_a
+    Calculate q_b
     E_c = E_a(1-q_a)^2+E_b(1-q_b)(1-q_a)^2
a    Continue
     STOP
```

Figure 6. An Algorithm for Computing E_c vs D_a

+ Subsection IV,2 addresses this problem.



A=2000	B=3000	D=10M	E=6000	K=1500
a=8000	b=106	k=0.5	r=0.5	s=106
M=6	N=9	I=14		

Figure 7. Plot of E_c vs. D_a

2. CALCULATION OF q_b IN THE TERRESTRIAL NETWORK

The calculation of q_b can be accomplished using the equations of the preceding subsection. A number of intimately linked variables are involved in transcendental relations. Consequently the sequence of steps in solving for q_b is important. We begin by rewriting (103) using (95), (99), and (104):

$$\frac{E_T'}{I\rho_L} = 1 - q_L \tag{110}$$

where

$$\rho_L = C_5(1-q_a)^2+C_6E_T' \tag{111}$$

$$C_L = C_7-C_8D_a \tag{112}$$

$$q_L = E_B(\rho_L, C_L) \; . \tag{113}$$

Now for some fixed values in the parameters (D_a, I,M,N), equation (110) can be written as three equations:

$$X(E_T') = E_T'/I\,\rho_L \tag{114}$$

$$Y(E_T') = 1-q_L \tag{115}$$

$$X(E_T^{*\prime}) - Y(E_T^{*\prime}) = 0 \; . \tag{116}$$

An analysis of (105) shows that, since q_b is bounded by zero and one, E_T' is bounded by

$$0 < E_T' \leq (1-q_a)^2/C_2 = \frac{(1-q_a)^2 EL(MN-1)}{M^2\,N^2} \; . \tag{117}$$

Hypothetical plots of $X(E_T^{\prime})$ and $Y(E_T^{\prime})$ are shown in Figure 8 and suggest the algorithm of Figure 9. After using the algorithm to find $E_T^{*\prime}$, then, we have, from (105) ·

$$q_b = 1-[C_2 E_T^{*\prime}/(1-q_a)^2]. \qquad (118)$$

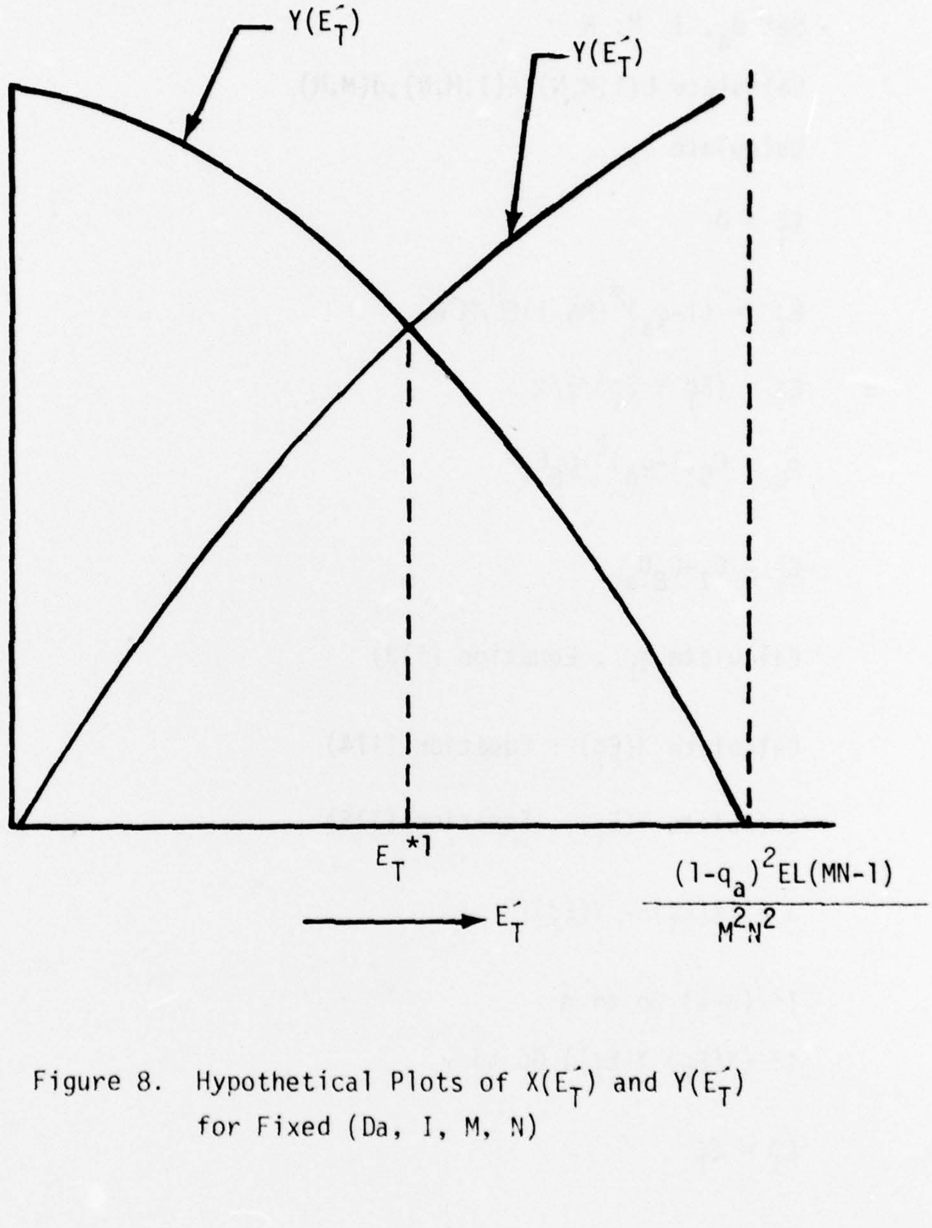


Figure 8. Hypothetical Plots of $X(E_T')$ and $Y(E_T')$ for Fixed (Da, I, M, N)

Read A, B, D, E, K, r, s, a, b, k, ε

Set D_a, I, M, N

Calculate $L(I,M,N)$,$\ell(I,M,N)$,$d(M,N)$

Calculate q_a

$E_T'' = 0$

$E_T''' = (1-q_a)^2(MN-1)EL/M^2N^2$

α $E_T' = (E_T'' + E_T''')/2$

$\rho_L = C_5(1-q_a)^2+C_6E_T'$

$C_L = C_7-C_8D_a$

Calculate q_L : Equation (113)

Calculate $X(E_T')$: Equation (114)

Calculate $Y(E_T')$: Equation (115)

$\delta = |X(E_T') - Y(E_T')|$

If $(\delta \leq \varepsilon)$ Go to β

If $(X(E_T')\ Y(E_T'))$ Go to γ

$E_T'' = E_T'$

Go to α

γ $E_T''' = E_T'$

Go to α

β $E_T^{*\prime} = E_T'$

STOP

Figure 9. An Algorithm for Computing $E_T^{*\prime}$

3. A CHECK ON THE SEARCH ALGORITHM FOR THE ALLOCATION PROBLEM

Subsection IV,1 gives an algorithm for a "brute force" technique for finding the optimum allocation of a total budget into access area and backbone designs; in fact, it gives the curve of E_c vs D_a (Figure 7) for a particular problem. We can now run this same problem through the more elegant algorithm of Figure 5 to check its accuracy. When this is done, we find the following optimum solution:

$D_a^* = 2,866,625$

$D_b^* = 7,133,375$

$q_a^* = 0.001$

$q_b^* = 0.001$

$q_L^* = 0.001$

$G^* = 0.004 \qquad (G = 1.-E_c/E)$

These results compare closely with those of Figure 7.

4. CALCULATING THE PERFORMANCE CHARACTERISTIC OF THE TERRESTRIAL NETWORK

Calculation of the performance characteristic requires, for each network design, the measurement of:

(a) Throughput, U_b, of the design under benign conditions. This is E_c, total carried traffic, in this report.

(b) Throughput, U_a, of the design <u>after</u> W backbone nodes have been removed from the design. This can be developed using the algorithms of this report.

Suppose then that we have an optimal network design characterized by (D_a^*, I^*, M, N). Removing W backbone nodes from this design results in two major effects;

(a) The backbone node grid is reduced to M' X N' where

$$M'N' = MN-W. \qquad (119)$$

A secondary effect of this is a change in the incidence degree, $I^{*'}$, of the remaining backbone nodes [2]:

$$I^{*'} = (MNI^* - 2WI^*)/(MN-W). \qquad (120)$$

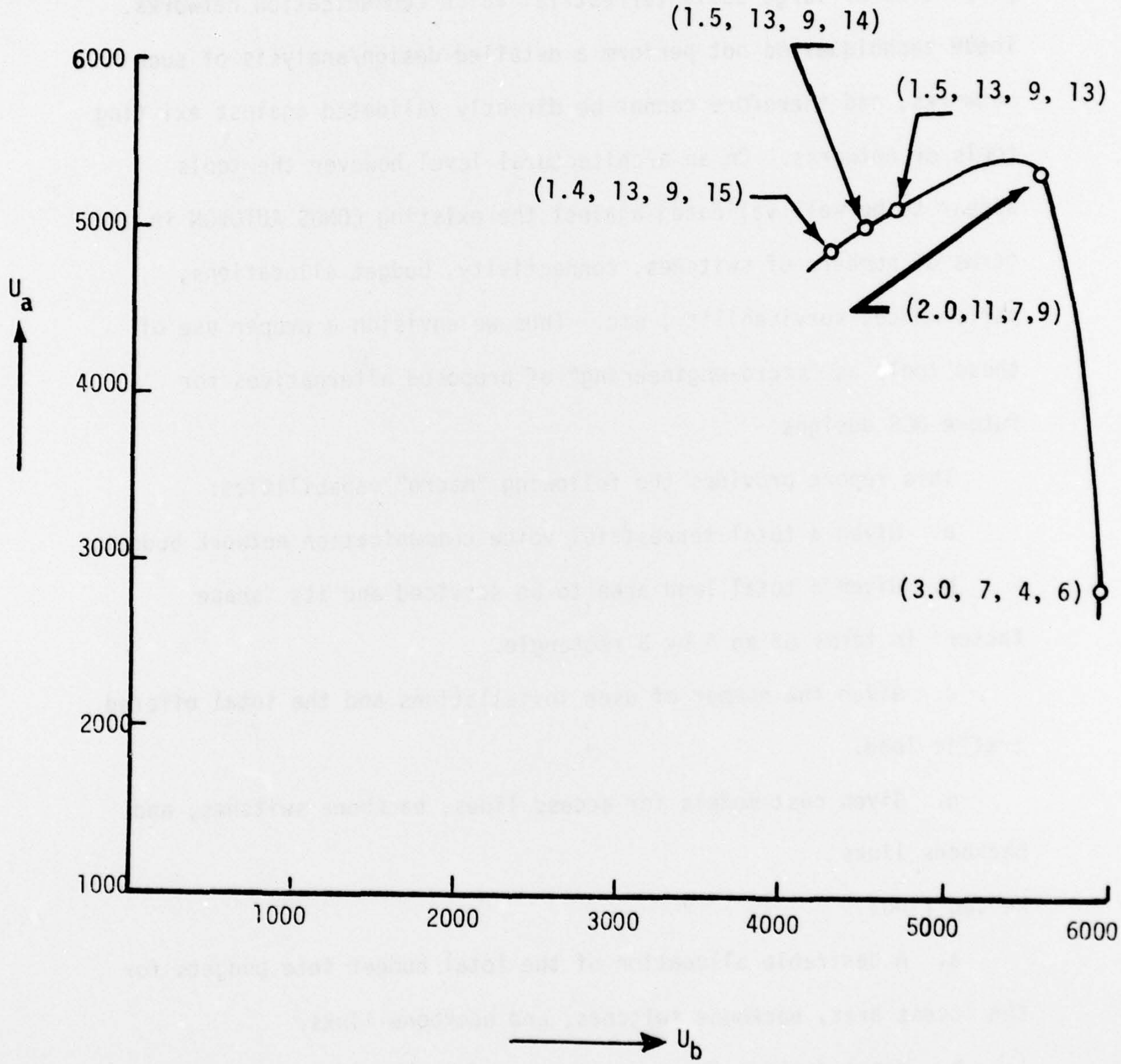
(b) A number of local switches (those homed on removed backbone nodes) are disconnected from each other and from the backbone. This reduces the total offered traffic, E', to

$$E' = EM'N'/MN. \qquad (121)$$

If we now repeat the analysis on the degraded network using the relations (119) - (121), we can calculate the network throughput, U_a, after the attack. The principal assumption required is:

A7. Each local switch that remains connected to the backbone network still offers E/K erlangs of traffic to the network. This offered traffic is equally likely to be destined for any of the local switches still connected to the backbone.

An example problem and results are given in Figure 10. As the figure shows, a point exists beyond which it is not advisable to increase the number of backbone switches (the maximum number of backbone switches would be K, one for each user installation; in this case there would be no "backbone"). We can also observe that optimum allocation of the budget depends on the mix of (U_b, U_a) desired.



A=2000 B=3000 D=8,000,000 E=6000 K=1500

a=8000 b=106 k=0.5 r=0.5 s=106

Figure 10. Performance Characteristic of a Terrestrial Network; W=9

V. CONCLUSIONS

This report has developed new techniques for analyzing average properties of large scale terrestrial voice communication networks. These techniques do not perform a detailed design/analysis of such networks, and therefore cannot be directly validated against existing tools or networks. On an architectural level however the tools appear to be well validated against the existing CONUS AUTOVON in terms of numbers of switches, connectivity, budget allocations, performance, survivability, etc. Thus we envision a proper use of these tools as "macro-engineering" of proposed alternatives for future DCS designs.

This report provides the following "macro" capabilities:

a. Given a total terrestrial voice communication network budget.

b. Given a total land area to be serviced and its "shape factor" in terms of an A by B rectangle.

c. Given the number of user installations and the total offered traffic load.

d. Given cost models for access lines, backbone switches, and backbone links .

We can find:

a. A desirable allocation of the total budget into budgets for the access area, backbone switches, and backbone links.

b. Macro designs for access area and backbone; in terms of

number of backbone switches, average connectivities and link capacities.

c. The performance and survivability characteristics attainable under various budget constraints and allocation options.

Since the tools are intended for macro analysis of future DCS alternatives we thus have a mechanism for rapidly varying and analyzing our assumptions (in terms of the givens) about the future. Thus we have a means for searching out alternatives which are robust under a wide range of future assumptions.

By using these tools to narrow down the extremely wide range of future alternatives to a robust few, and by providing macro designs for these, the work load on other elements in DCEC which do a very thorough and detailed engineering job on alternatives can be streamlined. Our next effort will be to extend the results herein to include terrestrial/satellite trade offs.

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